Poznań University of Technology. Electronics and Telecommunications
subject: *Selected topics in mathematics*, code: 1010802111010342874, 2^{nd} cycle, 1^{st} semester
45 lectures, 45 classes

Exam topics

A. General look at mathematics, incl.
   - an axiomatic structure of mathematics, a.o., that grounded on the set theory,
   - an equivalence relation, its equivalency/abstract classes and a quotient space (also that concerning Lebesgue-integrable functions),
   - a function and its types (injection, surjection, bijection),
   - an Euclidean geometry and other geometries,
   - an Euclidean space (a space eqiupped with the Euclidean distance; this can be thought as the distance we measure using a ruler, the radio distance),
   - multifunctions, e.g. a n-th root and a logarithm in a complex domain,
   - a distribution (aka a generalized function), e.g., a Dirac delta (aka an infitely great point impulse) which can be seen as the inverse Laplace transform of $L(s)=1$, the derivative of the Heaviside jump function, the distributional limit of the sequence $(\delta_\sigma)_{\sigma\to 0}$, $\delta_\sigma(x) := \exp(-(x/\sigma)^2)/(\sigma\cdot\sqrt{\pi})$.

B. Repetition and supplement in the field of vector and matrix algebra:
   - an anchored/bounded vector, a free vector,
   - a dot/scalar/inner product of vectors, its presence, e.g. in the Pearson correlation coefficient of two samples $y = (y_k)_{k=1..N}$ and $z = (z_k)_{k=1..N}$,

     $\mathrm{cocoP}(y,z) := \mathrm{cov}(y,z)/(\mathrm{std}(y)\cdot\mathrm{std}(z))$,

     where the covariance $\mathrm{cov}(y,z) := \Sigma_{k=1..N}\,(y_k-m_y)\cdot(z_k-m_z)/N$,

     the standard deviation $\mathrm{std}(y) := \sqrt{\mathrm{var}(y)}$,

     the variance $\mathrm{var}(y) := \mathrm{cov}(y,y)$,

     the mean $m_y := \Sigma_{k=1..N}\, y_k/N$,
   - 3D-vectors and their vector product,
   - a sale (system of algebraic linear equation) and its solvability,
   - an algebraic eigenproblem (eigenvalues and eigenvectors of a matrix),
   - the diagonalization of a matrix,
   - the Cayley-Hamilton theorem and $\exp(A)$, where A stands for a square matrix.

C. Polynomial collocation and least-square approximation. Theorems on the solvability.

D. Mathematics of the Fast Fourier transform.

E. Basic binary structures (grupoids, semigroups, monoids and groups, as well as their sub-, e.g., subgroups) and theorems on them (e.g., that on the uniqueness of a neutral element). Homomorphism and its types (mono-, epi- and isomorphism). Examples, a.o.,
   - $(\boldsymbol{Z}_n,+)$ – additive group of n-congruent integers, $(\boldsymbol{Z}_n,\cdot)$ – multiplicative monoids in the set of remainders obtained when integers are divided by a fixed natural $n > 1$,
   - a symmetric group: its mechanical realization, its permutation description (including the cyclic structure of a permutation, exhibited via the graph of a permutation), its matrix representation.

F. An (Euclidean) distance and a metric, a metric space. Examples.

G. A linear/vector space, a linear combination of its elements, a basis of a linear space (and a dimension of a linear space); linearl . The change of a basis. Examples of linear spaces (including infinite linear spaces).

H. A ring, examples (inc. the ring $\boldsymbol{R}_n[x]$ of polynomials in the variable x whose coefficients are real and the degree is $\leq n$).

I. A field, examples (incl. Galois fields, in particular GF(4)).

J. A norm and a normed space. Examples: vector norms, matrix norms, norms of functions.

K. A Banach space (a complete normed linear space, i.e., a normed space which is complete with the metric induced by the norm; the completeness stands for the following property: every Cauchy sequence is, in this metric, convergent to an element of the space at hand). Examples.

L. A scalar/inner product in a linear space, a space equipped with it (and called an unitary space, or pre-hilbertian space). Examples.

M. Orthogonality in an unitary space. Theorem on the orthogonal decomposition.

N. A Hilbert space (an unitary space, over a field $\boldsymbol{R}$ or $\boldsymbol{C}$, with the norm generated by an inner product and completed; equivalently: a Banach space with the norm induced by an inner product). Examples.

O. ODE1 (ordinary differential equation of the 1st order). Its power/Fröbenius solution (showing the presence of an arbitrary constant), particular cases (e.g., separable ODE1).

P. A general solution to a nonhomogeneous (or: inhomogeneous) ODE1:
GINE = GIHE plus PINE (general integral of nonhomogeneous eqn …)

Q. Standard methods to find solution/integral of an ODE1:
- a variation of a constant (in GIHE, aka a Duhamel method;
in Polish: MUS = metoda uzmienniania stałej),
- a search/look among possible shapes (aka a predicted shape method, a guess method;
in Polish: MOP = metoda oczekiwanej postaci).

R. Examples of ODE1, a.o.,
$y' = \lambda\cdot y$ – the exponential growth eqn, the eqn of the capacitor undergoing discharge,
$u' = \lambda\cdot(u - u_{\text{amb}})$ – the Newtonian cooling ($u=u(t)$ – the temperature at the moment t,
u_{amb} – the surrounding temperature),
$y' = a\cdot y - b\cdot y^2$ (equivalent to $u' = a\cdot u\cdot(1-u)$, $y = a/b\cdot u$) – the logistic/Verhulst eqn,
the bounded growth, exhibiting the saturation curve)

S. CC-HLODE2 (homogeneous linear ordinary differential eqn of the order 2 with constant coefficients), its role in the mechanics (mass-resistance-spring system) and the electricity (both parallel and serial RLC circuits). Its general solution produced by two linearly independent functions ($\exp(\lambda t)$ and $t\cdot\exp(\lambda t)$ if the characteristic equation has a double zero λ, $\exp(\lambda_1 t)$ and $\exp(\lambda_2 t)$ otherwise). The solution exhibiting trigonometric and hyperbolic functions.

T. HLODE2 with arbitrary coefficients, e.g., n-th
Legendre eqn $(1-x^2)\cdot y'' - 2x\cdot y' + n\cdot(n+1)\cdot y = 0$,
Chebychev eqn $(1-x^2)\cdot y'' - x\cdot y' + n^2\cdot y = 0$,
Laguerre eqn $x\cdot y'' + (1-x)\cdot y' + n\cdot y = 0$,
Hermite eqn $y'' - 2x\cdot y' + 2n\cdot y = 0$,
and their solutions (all orthogonal with corresponding weights in corresponding intrvals)

U. CC-HLPDE2 (homogeneous linear partial differential equation of the 2nd order with constant coefficients) in 2 variables, $Au_{xx} + 2Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = 0$, where $u = u(x,y)$, ux – the derivative of u wrt x etc., A, B, …, F – constant numbers such that $A^2+B^2+C^2 > 0$. The matrix form of this equation, its relation to the equation of a conic (laying in the Oxy plane) and the typology. The type recognition via eigenvalues of the associated matrix $[A, B; B, C]$.

V. Standard PDE2 (partial differential equation of the 2nd order) in two variables:
- of the elliptic type: Laplace eqn $\Delta u = 0$, i.e., $\nabla^2 u = 0$, i.e.,
$u_{xx} + u_{yy} = 0$, where $u = u(x,y)$,

– of the parabolic type: heat eqn $u_t = c\cdot u_{xx}$, where $u=u(t,x)$ – the temperature at the moment t and at the point x, c – a positive number referred to as a thermal diffusivity,
– of the hyperbolic type: string, or wave, eqn $u_{tt} = c^2\cdot u_{xx}$, where $u = u(t,x)$ – the displacement from the equilibrium at the moment t and at the point x, c – a positive number referred to as a propagation speed (of the move along the string)

W. Derivation of the string/wave equation:
- after Jean le Rond d'Alembert (*Recherches sur les cordes vibrantes* 1747, based on the equilibrium between Newton force and Hooke force).
- probabilistic derivation (referring to a random walk).

X. Solving a wave equation:
- d'Alembert solution: $u(t,x) = R(x-ct) + L(x+ct)$ – wave travelling with the speed c to the right and to the left,
- Fourier solution (based on the factorial assumption and Fourier composition): $u(t,x) = T(t)\cdot X(x)$, so $T''(t)\cdot X(x) = c^2\cdot T(t)\cdot X''(x)$ and there are two ODE2: $T''(t)/(c^2\cdot T(t)) = -\lambda$ and $X''(x)/X(x) = -\lambda$ with some constant λ, in consequence u is a series of Fourier-like terms.

Y. Examples of non-linear PDE:
- Klein-Gordon eqn: $\varphi_{tt} - \varphi_{xx} + \varphi = 0$ (Oscar Klein, Walter Gordon 1926; J.Frenkel, T.Kontorova 1939),
- sine-Gordon eqn: $\varphi_{tt} - \varphi_{xx} + \sin\varphi = 0$, with $u=(x+t)/2$, $v=(x-t)/2$ it is $\varphi_{uv} = \sin\varphi$ (Edmond Bour 1862),
- Korteweg-deVries eqn: $\phi_t - \phi_{xxx} - 6\phi\cdot\phi_x = 0$ (Joseph Boussinesq 1877, Diederik Kortenweg & Gustav de Vries 1895).

Exams take place on Monday, 26th of June, 9:45 for polish-language group, 11:30 for English-language group, on the next day students can see their answers between 10 and 14 (Polish group), 14 till 16 (English group).
The retake term is on Thursday 28th of September, 2017 at 9:45 (Polish-laguage group) and 11:30 (English language group).
The retake term is for who did not pass positively the exam in the first term or did not come to pass it (in this situation the first term is lost and the student has no more than one trial to gain a credit for the lecture course).
In the first term there will be offered 5 problems and a student passing the exams choses three of them, only answers to these three problems are checked. The maximum number of scores is 27 (= 3·9) and the numer of scores is transposed to the final mark as follows:

00-12: 2.0; 13-15: 3.0; 16-18: 3.5; 19-21: 4.0; 22-24: 4.5; 25-27: 5.0.

A student who showed to be active during the lectures may be presented with up to 3 additional scores.
Due to differences between programs realized in various countries, a student may resign from the exam if presents appropriate verification/certificate/transcript; in this case he/she is obliged first to copy (on his/her own sheet of paper) the exam problems, next to clearly state: 'Take into account, please, that I already was familiarized with topics presented in the course at hand and I passed positively a required exam, as it is stated in the certificate which copy hereby I pass to you'; then I approve the course I did with the lowest positive mark (3.0, intern. E).
The mark is sent to the student via eProto system in the evening of 27th of June.
No later than on 16th of July, 2017, a student may find me in my office (El-744) during my service hour (pleas, consult them in the internet page), may raise objections to the issued scoring and, finally, may be examined orally (but only if in the written exam got at least 10 scores); during this oral exam/talk, depending on the quality of answers, the mark can be kept, can be raised up or can be verified for a lower one.

Regulations for the retake exam (2017.09.28) are analogical, e.g., a student chooses 2 out of 4 proposed problems, the transfer from gathered scores (the maximum is 2·9 = 18) is as follows **0-7: 2.0; 8-9: 3.0; 10-11: 3.5; 12-13: 4.0; 14-15: 4.5; 16-18: 5.0**, an additional question (in an oral form) is proposed only to whom obtained at least 5 points in the written part, an oral exam may keep the mark already issued, may it raise up or may it take down.

Adam Marlewski, 2015-05-25, 16:21